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Two-particle kinematic distributions from new physics at an electron–positron collider with polarized beams

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Abstract. The kinematic distributions in two-particle inclusive processes at an e^+e^- collider arising from standard-model s-channel exchange of a virtual γ or Z and the interference of the standard-model contribution with contributions from physics beyond the standard model involving s-channel exchanges are derived entirely in terms of the space-time signature of such new physics. Transverse as well as longitudinal polarizations of the electron and positron beams are taken into account. We show how these model-independent distributions can be used to deduce some general properties of the nature of the interaction. We then specialize to two specific two-particle final states, viz., ZH, where H is one of the Higgs bosons in a model with an extended Higgs sector, and $f\overline{f}$, where f, $\overline{f}$ are a pair of conjugate charged fermions, wherein distributions of two (of the possibly several) decay products are measured. We show how some of the properties of the distributions have been realized in the analysis of physics beyond the standard model in earlier work which made use of two-particle angular distributions.

1 Introduction

The proposed International Linear Collider (ILC) which could collide e^+ and e^- at a centre of mass energy of several hundred GeV, if built, would serve as an instrument for precision measurements of various parameters underlying particle physics [1]. A strong beam polarization programme of transverse or longitudinal beam polarization is also being seriously proposed by investigators in the field [2, 3]. Besides carrying out precision studies of properties of standard-model (SM) particles, the ILC is geared to probe physics beyond the standard model (BSM). In particular, the ILC will be sensitive to BSM physics even if the energy is not sufficient to produce BSM particles directly, via precision studies that are sensitive to the propagation of such particles in loops. BSM physics can manifest itself in a variety of new effects including CP violation, momentum correlations of SM particles, correlations involving spins of decaying particles as well as spins of the electron and positron beams.

Recently, we presented an approach that relies on the characterization of new physics in terms of its space-time transformation properties [4] using one-particle inclusive distributions in e^+e^- annihilation. This approach was model independent, and relied only on the use of Lorentz covariance for deriving the most general form of one-particle kinematic distributions for the cases where the BSM interaction had different space-time properties. The distributions were expressed in terms of Lorentz-invariant 'structure functions' much as kinematic distributions in deep-inelastic scattering are characterized in terms of structure functions in what is now standard treatment. We demonstrated the utility of such an approach for a general analysis of different types of processes.

For processes where the single particle of interest is heavy, it is unstable and is therefore invariably detected by means of its decay products. Thus, the process then has at least two particles in the final state whose momenta are measured, and an extension to two-particle distributions would be useful. In addition to this motivation, a two-particle inclusive distribution would carry more information than a one-particle one. In fact, there can be qualitatively new information in the two-particle case. For instance, as we will see later, in certain situations, signatures of CP violation are absent in the one-particle exclusive final case, but appear naturally in the two-particle case. Keeping these motivations in mind, we have now extended our study to the two-particle inclusive process $e^+e^- \to h_1(p_1)h_2(p_2)X$, where h_1 and h_2 denote two SM particles that are detected, and p_1 and p_2 are their respective momenta. The latter process is depicted in Fig. 1. It may be noted that this general process actually encompasses a class of different exclusive processes, including those where h_1 and h_2 arise from the decay of a heavy particle or a resonance.

In spirit it is the extension of the pioneering work of Dass and Ross [5, 6] that had been performed in the context of γ

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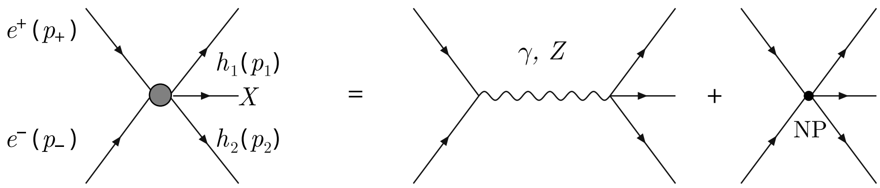

Fig. 1. The basic process

contributing to the s-channel production, probing the then undiscovered neutral current. Our work in practice is the inclusion of Z in the s-channel, in addition to γ, and where now it is the as-yet undiscovered new physics that we intend to probe. Significant new features arise due to the presence of the axial-vector coupling of the Z to the electron, a feature missing in a vector theory like QED. Indeed, a significant feature is that here too there are additional structure functions compared to the analysis presented in [4]. These are absent in the case of the reaction where only one particle was detected with no spin information. It may also be emphasized that once a general discussion is provided for an inclusive final state, it may be readily applied to exclusive final states as well, thereby providing a framework for studying several processes of interest. More importantly, the inclusive state could simply arise from the decays of the particles in a two-particle final state.

Our formalism is restricted to the broad framework originally utilized in [5, 6], which envisages new physics only through an s-channel exchange. Thus the BSM effects could arise through the exchange of a new particle like a new gauge boson Z', or through the exchange of Z, but a with a BSM vertex or a SM loop producing the final state in question. Moreover, the SM contribution is also assumed to be through the tree-level exchange of a virtual photon and a virtual Z. There are other interesting scenarios where the present formalism is not applicable without modification. For example, in the production of a SM gauge-boson pair in the final state, the SM contribution is via a t-channel exchange of an electron, not amenable to description in the present formalism. However, suitable modifications of our approach, to be pursued in future, suggest themselves to deal with such situations. A modified formalism for the γZ final state was discussed in [4], and the results compared with those in [7–10].

The expectations from our general model-independent analysis is shown for some specific processes to be consistent with the results obtained earlier for those processes. Our approach would thus be useful to derive general results for newer processes which fall within the framework described above. We would thus be able to anticipate certain results without a detailed calculation for each individual process.

In Sect. 2 we present a computation of the spin-momentum correlations resulting from the presence of structure functions that characterize the new physics. Our results here are presented in the form of results arising from the computation of a trace that encodes the leptonic tensor as well as the new physics encoded in a tensor constructed out of the momenta of the observed final-state particles (what is known as a 'hadronic' tensor, for historical reasons, since the term arose at a time when the final state consisted largely of hadrons). These tables provide the analogue for the SM and new physics, of what was provided by Dass and Ross [6] for QED and neutral currents. In Sect. 3 we discuss the CP and T properties of correlations for different classes of inclusive and exclusive final states. In Sect. 4 we provide a discussion on the the polarization dependence of the correlations in different cases. In Sect. 5 we will specialize to two specific examples of processes, into which our approach can give significant insight. In Sect. 6 we present our conclusions and discuss prospects for extension of the present framework to account for classes of BSM interactions not presently covered.

2 Computation of correlations

We consider the two-particle inclusive process

$$e^-(p_-) + e^+(p_+) \to h_1(p_1) + h_2(p_2) + X\,, \qquad (1)$$

where h_1 and h_2 are final state particles whose momenta are measured, but not their spin, and X is an inclusive state. The process is assumed to occur through an s-channel exchange of a photon and a Z in the SM, and through a new current whose coupling to e^+e^- can be of the type V, A, or S, P, or T.

Since we will deal with a general case without specifying the nature or couplings of h_1, h_2, we do not attempt to write the amplitude for the process (1). We will only obtain the general form, for each case of the new coupling, of the contribution to the angular distribution of h_1, h_2 from the interference of the SM amplitude with the new physics amplitude.

It might be clarified here that even though we use the term "inclusive" implying that no measurement is made on the state X, in practice it may be that the state X is restricted to a concrete one-particle or two-particle state which is detected. In such a case the sum is not over all possible states X. Nevertheless, the momenta of the few particles in the state X are assumed to be integrated over, so that there is a gain in statistics as compared to a completely exclusive measurement. The angular distributions we calculate hold also for such a case, except that structure functions would depend on the states included in X.

Following Dass and Ross [5, 6], we calculate the relevant factor in the interference between the standard model currents with the BSM currents as

$$\begin{aligned}&\mathrm{Tr}\left[(1-\gamma_5 h_+ + \gamma_5 \not{s}_+)\not{p}_+\gamma_\mu\left(g_\mathrm{V}^e - g_\mathrm{A}^e\gamma_5\right)\right.\\ &\quad\left.\times(1+\gamma_5 h_- + \gamma_5\not{s}_-)\not{p}_-\Gamma_i\right]H^{i\mu}\,. \qquad (2)\end{aligned}$$

Here $g_\mathrm{V}^e, g_\mathrm{A}^e$ are the vector and axial-vector couplings of the photon or Z to the electron current, and Γ_i is the corresponding coupling to the new-physics current, $h_\pm$ are the helicities (in units of $\frac{1}{2}$) of $e^\pm$, and $s_\pm$ are respectively their transverse polarizations. For ease of comparison, we have sought to stay with the notation of [5, 6], with some exceptions which we spell out when necessary. We should of course add the contributions coming from photon exchange and Z exchange, with the appropriate propagator factors. However, we give here the results for Z exchange, from which the case of photon exchange can be deduced as a special case. The tensor $H^{i\mu}$ stands for the interference between the couplings of the final state to the SM current and the new-physics current, summed over final-state polarizations, and over the phase space of the unobserved particles X. It is only a function of the the momenta $q = p_- + p_+$, p_1 and p_2. The implied summation over i corresponds to a sum over the forms V, A, S, P, T, together with any Lorentz indices that these may entail.

We now determine the forms of the matrices Γ_i and the tensors $H^{i\mu}$ in the various cases, using only Lorentz covariance properties. Our additional currents are as in [5, 6], except for the sign of g_A in the following. We explicitly note that in our convention is $\epsilon^{0123} = +1$.[1] We set the electron mass to zero. Consider now the three cases:

2.1 Scalar and pseudoscalar case

In this case, there is no free Lorentz index for the leptonic coupling. Consequently, we can write it as

$$\Gamma = g_\mathrm{S} + \mathrm{i} g_\mathrm{P}\gamma_5 \,. \tag{3}$$

The tensor $H^{i\mu}$ for this case has only one index, viz., μ. Hence the most general form for H is

$$H^\mathrm{S}_\mu = \left(r_\mu - q_\mu \frac{r\cdot q}{q^2}\right) F^r \,, \tag{4}$$

where r is p_1, p_2 or n ($n_\rho \equiv \epsilon_{\rho\alpha\beta\gamma} p_1^\alpha p_2^\beta q^\gamma$).

2.2 Vector and axial-vector case

The leptonic coupling for this case can be written as

$$\Gamma_\nu = \gamma_\nu (g_\mathrm{V} - g_\mathrm{A}\gamma_5) \,. \tag{5}$$

Note that we differ from Dass and Ross [5, 6] in the sign of the g_A term. The tensor H for this case has two indices, and can be written as

$$\begin{aligned} H^V_{\mu\nu} &= -g_{\mu\nu} W_1 + \frac{1}{2}(r_\mu t_\nu + r_\nu t_\mu) W_2^{rt} \\ &\quad + \epsilon_{\mu\nu\alpha\beta} u^\alpha v^\beta W_3^{uv} + \frac{1}{2}(p_{1\mu} p_{2\nu} - p_{1\nu} p_{2\mu}) W_4 \,, \end{aligned} \tag{6}$$

where $W_1, W_2^{rt}, W_3^{uv}, W_4$ are invariant functions, and r, t can be chosen from p_1, p_2 and n, and u, v can be chosen from p_1, p_2 and q. As compared to the one-particle exclusive case, there is an additional tensor structure with structure function W_4, which requires two particles, being antisymmetric in p_1 and p_2.

2.3 Tensor case

In the tensor case, the leptonic coupling is

$$\Gamma_{\rho\tau} = g_\mathrm{T} \sigma_{\rho\tau} \,. \tag{7}$$

The tensor H for this case can be written in terms of the four invariant functions F_1, F_2, PF_1, PF_2 as

$$\begin{aligned} H^T_{\mu\rho\tau} &= (r_\rho u_\tau - r_\tau u_\rho) t_\mu F_1^{rut} + (g_{\rho\mu} r_\tau - g_{\tau\mu} r_\rho) F_2^r \\ &\quad + \epsilon_{\rho\tau\alpha\beta} r^\alpha u^\beta t_\mu P F_1^{rut} + \epsilon_{\rho\tau\mu\alpha} r^\alpha P F_2^r \,, \end{aligned} \tag{8}$$

where t is chosen from p_1, p_2 and n, u from p_1, p_2, q and n, r from p_1, p_2 and q. These choices of vectors for r, t, and u give a complete set of independent tensors. The use of vectors other than covered by the choices would result in tensors which are combinations of tensors described by (8). Details can be found in [6].

The structure functions introduced in the above are functions of the Lorentz invariants that can be formed from the momenta q, p_1 and p_2. The dependence of these functions on the Lorentz invariants encode the dynamics of the BSM interactions. In particular, they would contain propagators and form factors occurring in the BSM amplitudes. We next substitute the leptonic vertices Γ and the respective tensors H^i in (2), and evaluate the trace in each case. We present the results in Tables 1–3, with $\vec{K} \equiv (\vec{p}_- - \vec{p}_+)/2 = E\hat{z}$, where $\hat{z}$ is a unit vector in the z-direction, E is the beam energy, and $\vec{s}_\pm$ lie in the x–y plane. A superscript T on a vector is used to denote its component transverse with respect to the e^+e^- beam directions. For example, $\vec{r}^\mathrm{T} = \vec{r} - \vec{r}\cdot\hat{z}\hat{z}$, and similarly for other vectors. The tables include, in addition to results presented in [5, 6] with only the g_V^e coupling relevant for QED, also those with g_A^e couplings, relevant for Z exchange in SM. Tables 1, 2 and 3 are respectively for cases of scalar–pseudoscalar, vector–axial-vector and tensor couplings respectively.

In the case of S, P and T couplings, all the entries in the corresponding tables vanish for unpolarized beams, or for longitudinally polarized beams. This is because there is no interference between the SM contribution, where the coupling to e^+e^- is of the V, A (chirality-conserving) type, and the contributions with S, P or T couplings, which are chirality violating. Thus at least one beam has to be

Table 1. List of S and P correlations

Term	Correlation
$\mathrm{Im}(g_\mathrm{P} F^r)$	$2E^2\vec{r}\cdot[-g_V^e(\vec{s}_+ - \vec{s}_-) + g_A^e(h_+\vec{s}_- + h_-\vec{s}_+)]$
$\mathrm{Im}(g_\mathrm{S} F^r)$	$2E\vec{K}\cdot[g_V^e(\vec{s}_+ + \vec{s}_-) + g_A^e(h_+\vec{s}_- - h_-\vec{s}_+)]\times\vec{r}$
$\mathrm{Re}(g_\mathrm{S} F^r)$	$2E^2\vec{r}\cdot[g_A^e(\vec{s}_+ + \vec{s}_-) + g_V^e(h_+\vec{s}_- - h_-\vec{s}_+)]$
$\mathrm{Re}(g_\mathrm{P} F^r)$	$2E\vec{K}\cdot[g_A^e(\vec{s}_+ - \vec{s}_-) - g_V^e(h_+\vec{s}_- + h_-\vec{s}_+)]\times\vec{r}$

[1] It may be noted that the convention actually used in [4] was $\epsilon^{0123} = +1$, with which all the results presented there being self-consistent. The corresponding remark made there about the convention in [5, 6] may be disregarded.

Table 2. List of V and A correlations

Term	Correlation
$\mathrm{Re}(g_{\mathrm{V}} W_1)$	$4E^2[g^e_{\mathrm{A}}(h_+ - h_-) - g^e_{\mathrm{V}}(h_+h_- - 1)]$
$\mathrm{Re}(g_{\mathrm{A}} W_1)$	$4E^2[g^e_{\mathrm{V}}(h_+ - h_-) - g^e_{\mathrm{A}}(h_+h_- - 1)]$
$\mathrm{Re}(g_{\mathrm{V}} W_2^{rt})$	$2E^2\{g^e_{\mathrm{A}}(h_+ - h_-)\vec{r}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}} - g^e_{\mathrm{V}}[\vec{r}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}}(h_+h_- - 1 - \vec{s}_+\cdot\vec{s}_-) + (\vec{r}\cdot\vec{s}_-)(\vec{t}\cdot\vec{s}_+) + (\vec{r}\cdot\vec{s}_+)(\vec{t}\cdot\vec{s}_-)]\}$
$\mathrm{Re}(g_{\mathrm{A}} W_2^{rt})$	$2E^2\{g^e_{\mathrm{V}}(h_+ - h_-)\vec{r}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}} - g^e_{\mathrm{A}}[\vec{r}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}}(h_+h_- - 1 + \vec{s}_+\cdot\vec{s}_-) - (\vec{r}\cdot\vec{s}_-)(\vec{t}\cdot\vec{s}_+) - (\vec{r}\cdot\vec{s}_+)(\vec{t}\cdot\vec{s}_-)]\}$
$\mathrm{Im}(g_{\mathrm{A}} W_2^{rt})$	$Eg^e_{\mathrm{V}}[(\vec{s}_-\cdot\vec{t})\vec{r}\cdot(\vec{K}\times\vec{s}_+) + (\vec{s}_+\cdot\vec{t})\vec{r}\cdot(\vec{K}\times\vec{s}_-) + (\vec{s}_-\cdot\vec{r})\vec{t}\cdot(\vec{K}\times\vec{s}_+) + (\vec{s}_+\cdot\vec{r})\vec{t}\cdot(\vec{K}\times\vec{s}_-)]$
$\mathrm{Im}(g_{\mathrm{V}} W_2^{rt})$	$-Eg^e_{\mathrm{A}}[(\vec{s}_-\cdot\vec{t})\vec{r}\cdot(\vec{K}\times\vec{s}_+) + (\vec{s}_+\cdot\vec{t})\vec{r}\cdot(\vec{K}\times\vec{s}_-) + (\vec{s}_-\cdot\vec{r})\vec{t}\cdot(\vec{K}\times\vec{s}_+) + (\vec{s}_+\cdot\vec{r})\vec{t}\cdot(\vec{K}\times\vec{s}_-)]$
$\mathrm{Im}(g_{\mathrm{V}} W_3^{uv})$	$4E^2(-v^0u^3 + v^3u^0)[-g^e_{\mathrm{V}}(h_+ - h_-) + g^e_{\mathrm{A}}(h_+h_- - 1)]$
$\mathrm{Im}(g_{\mathrm{A}} W_3^{uv})$	$4E^2(-v^0u^3 + v^3u^0)[-g^e_{\mathrm{A}}(h_+ - h_-) + g^e_{\mathrm{V}}(h_+h_- - 1)]$
$\mathrm{Im}(g_{\mathrm{V}} W_4)$	$2E(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_+[g^e_{\mathrm{V}}(h_+ - h_-) - g^e_{\mathrm{A}}(h_+h_- - 1)]$
$\mathrm{Im}(g_{\mathrm{A}} W_4)$	$2E(\vec{p}_1\times\vec{p}_2)\cdot\vec{p}_+[g^e_{\mathrm{A}}(h_+ - h_-) - g^e_{\mathrm{V}}(h_+h_- - 1)]$

Table 3. List of T correlations

Term	Correlation
$\mathrm{Im}(g_{\mathrm{T}} F_1^{rut})$	$4E^2\{[(\vec{r}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}})\vec{u} - (\vec{u}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}})\vec{r}]\cdot[g^e_{\mathrm{A}}(\vec{s}_+ - \vec{s}_-) - g^e_{\mathrm{V}}(h_-\vec{s}_+ + h_+\vec{s}_-)]$ $+(r^0u^3 - r^3u^0)\vec{t}\cdot[g^e_{\mathrm{A}}(\vec{s}_+ + \vec{s}_-) - g^e_{\mathrm{V}}(h_-\vec{s}_+ - h_+\vec{s}_-)]\}$
$\mathrm{Im}(g_{\mathrm{T}} F_2^{r})$	$4E^2\vec{r}\cdot[-g^e_{\mathrm{A}}(\vec{s}_+ - \vec{s}_-) + g^e_{\mathrm{V}}(h_-\vec{s}_+ + h_+\vec{s}_-)]$
$\mathrm{Im}(g_{\mathrm{T}} PF_1^{rut})$	$4E\{E[r^0(\vec{u}\times\vec{t}^{\,\mathrm{T}}) - u^0(\vec{r}\times\vec{t}^{\,\mathrm{T}})]\cdot[-g^e_{\mathrm{A}}(\vec{s}_+ - \vec{s}_-) + g^e_{\mathrm{V}}(h_-\vec{s}_+ + h_+\vec{s}_-)]$ $-\{[(\vec{r}\times\vec{u})\cdot\vec{p}_+]\vec{t}\cdot[-g^e_{\mathrm{A}}(\vec{s}_+ + \vec{s}_-) + g^e_{\mathrm{V}}(h_-\vec{s}_+ - h_+\vec{s}_-)]\}$
$\mathrm{Im}(g_{\mathrm{T}} PF_2^{r})$	$4E[-g^e_{\mathrm{A}}(\vec{s}_+ + \vec{s}_-) + g^e_{\mathrm{V}}(h_-\vec{s}_+ - h_+\vec{s}_-)]\times\vec{K}\cdot\vec{r}$
$\mathrm{Re}(g_{\mathrm{T}} F_1^{rut})$	$4E(E[r^0(\vec{u}\times\vec{t}^{\,\mathrm{T}}) - u^0(\vec{r}\times\vec{t}^{\,\mathrm{T}})]\cdot[-g^e_{\mathrm{V}}(\vec{s}_+ + \vec{s}_-) + g^e_{\mathrm{A}}(h_-\vec{s}_+ - h_+\vec{s}_-)]$ $+\{[(\vec{r}\times\vec{u})\cdot\vec{p}_+]\vec{t}\cdot[-g^e_{\mathrm{V}}(\vec{s}_+ - \vec{s}_-) + g^e_{\mathrm{A}}(h_-\vec{s}_+ + h_+\vec{s}_-)]\})$
$\mathrm{Re}(g_{\mathrm{T}} F_2^{r})$	$4E[-g^e_{\mathrm{V}}(\vec{s}_+ - \vec{s}_-) + g^e_{\mathrm{A}}(h_-\vec{s}_+ + h_+\vec{s}_-)]\times\vec{K}\cdot\vec{r}$
$\mathrm{Re}(g_{\mathrm{T}} PF_1^{rut})$	$-4E^2\{[(\vec{r}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}})\vec{u} - (\vec{u}^{\,\mathrm{T}}\cdot\vec{t}^{\,\mathrm{T}})\vec{r}]\cdot[g^e_{\mathrm{V}}(\vec{s}_+ + \vec{s}_-) - g^e_{\mathrm{A}}(h_-\vec{s}_+ - h_+\vec{s}_-)]$ $+(r^0u^3 - r^3u^0)\vec{t}\cdot[g^e_{\mathrm{V}}(\vec{s}_+ - \vec{s}_-) - g^e_{\mathrm{A}}(h_-\vec{s}_+ + h_+\vec{s}_-)]\}$
$\mathrm{Re}(g_{\mathrm{T}} PF_2^{r})$	$4E^2\vec{r}\cdot[g^e_{\mathrm{V}}(\vec{s}_+ + \vec{s}_-) - g^e_{\mathrm{A}}(h_-\vec{s}_+ - h_+\vec{s}_-)]$

transversely polarized to see the interference. Also, in these cases, it is sufficient to have either e^- or e^+ beams transversely polarized – it is not necessary for both beams to have transverse polarization. However, to observe terms which correspond to combinations like $(h_-\vec{s}_+ \pm h_+\vec{s}_-)$, it is necessary to have at least one beam longitudinally polarized, and the other transversely polarized. Considering that such a configuration, though feasible, is not a simple option from the experimental point of view, in the practical case when one beam or both beams are transversely polarized, it can be seen from Tables 1 and 3 that it is only the coupling g^e_{V} which goes with the imaginary part of the structure functions in case of S, P couplings, and g^e_{A} in case of T couplings. Likewise, g^e_{A} and g^e_{V} occur with the real parts in these respective cases.

In case of V and A couplings, both beams have to be polarized, or the effect of polarization vanishes. It is interesting to note that all the correlations in the latter case are symmetric under the interchange of $\vec{s}_+$ and $\vec{s}_-$.

3 CP and T properties of correlations

It is important to characterize the C, P and T properties of the various terms in the correlations, which would in turn depend on the corresponding properties of the structure functions which occur in them.

In this context we recall that a similar analysis was done for the one-particle inclusive case treated in [4]. In that case, we deduced the important result that when the final state consists of a particle and its anti-particle, it is not possible to have any CP-odd term in case of V and A BSM interactions. This deduction depended on the property that in the centre-of-mass frame, the particle and anti-particle three-momenta are equal and opposite. In the present case of two-particle inclusive distributions, even if the two particles observed are conjugates of each other, their momenta are not constrained. Thus it is possible to have CP-odd correlations even in the V, A case.

We now come to a more systematic analysis. We consider two important cases, one when the particles h_1 and h_2 in the final state in $e^+e^- \to h_1h_2X$ are their own conjugates, and the other when they are not. We treat these two cases separately.

3.1 Case A: $h_1^c = h_1$, $h_2^c = h_2$

In this case, the first entry in Table 1 is CP even for $\vec{r} \equiv \vec{p}_1, \vec{p}_2$, that is, when $\vec{r}$ is an ordinary (polar) vector, and CP odd for $\vec{r} \equiv \vec{n}$, a pseudo-vector (axial vector). The same is true for the fourth entry in Table 1. On the other hand, the second and third entries are CP odd for $\vec{r} \equiv \vec{p}_1, \vec{p}_2$ and CP even for $\vec{r} \equiv \vec{n}$. Moreover, it can be checked that the first two entries are CPT even, where T is naïve time reversal, and the last two entries are CPT odd. (Henceforth, whenever we refer to T, we will mean naïve time reversal). This implies, via the CPT theorem, that the last two entries need the absorptive part to be non-zero.[2]

In Table 2, the entries with W_1 and those with W_4 are CP even, the former being even under naïve T and the latter odd. Of the other entries, the terms corresponding to W_2^{rt} are CP even when both $\vec{r}$ and $\vec{t}$ are ordinary vectors, and CP odd when one of $\vec{r}$ and $\vec{t}$ is an ordinary vector and the other is a pseudo-vector. Of these four rows in Table 2, the first two are T even and the next two T odd for the cases when these are all CP even, and the opposite T properties hold when they are all CP odd. The entries with W_3^{uv} are CP odd and T even for all u, v. In this case of V and A couplings, we see that terms which are even under CPT occur with the real part of the structure function, whereas those which are odd come with the imaginary part of the structure function.

In Table 3, the entries corresponding to F_1^{rut} and F_2^r are CP even when $\vec{u}$, $\vec{t}$ are ordinary vectors (since $\vec{r}$ can only be an ordinary vector), but CP odd when one of $\vec{u}$, $\vec{t}$ is a pseudo-vector. On the contrary, the entries corresponding to PF_1^{rut} and PF_2^r are CP odd when $\vec{u}$, $\vec{t}$ are ordinary vectors and CP even when one of $\vec{u}$, $\vec{t}$ is a pseudo-vector. Deductions regarding the presence or absence of absorptive parts follows on use of the CPT theorem after noting that when all vectors are ordinary vectors, the first two entries are T even, the next four are T odd, and the last two are again T even.[3]

3.2 Case B: $h_1^c \neq h_1$, $h_2^c \neq h_2$

In the case when h_1 and h_2 are not self-conjugate, the above statements under case A about the CP properties would be true for even linear combinations of the structure functions for production of h_1 and h_2 with the structure functions for the production of the conjugates of h_1 and h_2. For the odd linear combinations, the opposite CP properties would hold.

A special case worth considering is when h_1 and h_2 are conjugates of each other. In that case, one can decompose each term into a part which is even under interchange of the four-vectors p_1 and p_2, keeping in mind that the structure functions are functions of the invariants p_1^2, p_2^2, $p_1 \cdot p_2$, $q \cdot (p_1 + p_2)$, and q^2, which are even under interchange of p_1 and p_2, and of $q \cdot (p_1 - p_2)$, which is odd under that interchange. One has now to go through the previous analysis done in the case when the final-state particles were self-conjugate, resulting in somewhat different results. In general, there would again be different combinations of structure functions which would contribute CP even and CP odd terms. However, in some cases, the existing terms transform into themselves, with a factor of ± 1. In this case, the CP property is the same or opposite to that in the case of self-conjugate final state.

4 The effect of beam polarization

We now make some general deductions from the tables on the dependence of distributions on beam polarization. We include in this discussion the correlations obtained in [4] for the one-particle inclusive process as well. To make the discussion in this section self-contained we repeat some observations already made in Sect. 2.

The first observation that one can make is that the interference of the scalar/pseudoscalar and tensor BSM interactions with the SM contribution cannot be studied unless the electron and/or positron beams are polarized. Not only that, it is not sufficient to have longitudinal polarization. A nonzero transverse polarization is needed to observe the interference terms. This is easy to understand – in the limit of vanishing electron mass, the scalar and tensor couplings are chirality violating, whereas the vector and axial-vector SM couplings are chirality conserving. Thus, the two do not interfere, even for arbitrary longitudinal polarization.

The interference of the vector and axial-vector BSM contributions with the SM contributions, on the other hand, is nonzero for unpolarized beams as well as polarized beams. In the case of transverse polarization, it is necessary for both electron and positron beams to be polarized for a nonzero answer, in contrast to the case of scalar and tensor BSM interactions, where either electron or positron beam had to be polarized.

The second observation in the case of vector and axial-vector BSM interactions is that the structure functions which contribute when polarization is included are the same as the ones which contribute when beams are unpolarized, provided absorptive parts are neglected. We assume here that the final-state particles which are observed are themselves eigenstates of CP, in which case, the imaginary parts of the structure functions contain absorptive parts of the BSM amplitudes. In other words, no qualitatively new information is contained in the polarized distributions if we neglect the imaginary parts of the structure functions. This observation, which here is for a general s-channel BSM process, was made in the context of the process $e^+e^- \to HZ$ in [12] with anomalous γZH and ZZH vertices, and confirmed in [13] for general e^+e^-HZ contact interactions. Such an observation was also made in an older context in [14] for the process $e^+e^- \to 3\,\text{jets}$.

This observation is important because most BSM interactions are chirality conserving in the limit of massless

[2] For a review, see [11]. Normally, such an absorptive part would simply be indicated by the occurrence of the imaginary part, rather than the real part, of the relevant structure function. However, the definition of F^r in (4) needs an extra factor of i for this to happen. Such an i would be natural if the F^r were defined to be real for point couplings. However, we have maintained the definitions of [6].

[3] As noted in the previous footnote, but for the unfortunate absence of factors of i in the definition of each of the tensor structure functions in (7), the CPT-even entries would be associated with real parts of structure functions, and the CPT-odd entries with the imaginary parts.

electrons, and can therefore be cast in the form of vector and axial-vector couplings. Thus, in a large class of contexts and theories, it is possible to conclude that polarization does not give qualitatively new information, unless absorptive parts are involved. This argument can be turned around, and it is possible to conclude that polarization can be used to get information on absorptive parts of structure functions of BSM interactions, which cannot be obtained with only unpolarized beams.

As a caveat, we note the following: It should not be construed that polarization does not play any positive role for chirality-conserving interactions even when there are no absorptive parts. In various case, it is possible to enhance the sensitivity to BSM interactions with a judicious choice of signs of the polarization. Thus even when no new structure functions are uncovered by polarization, the information on structure functions which can be obtained with polarized beams can be quantitatively better than that obtained with unpolarized beams.

In our case, if absorptive parts are included, there is a contribution from $\mathrm{Im}\, W_3$ for the one-particle inclusive case considered in [4], and $\mathrm{Im}\, W_3^{uv}$ in the two-particle inclusive case discussed in this paper. Again, in this case, it possible to predict the differential cross section for the polarized case, if the unpolarized cross section is known.

On the other hand, we see that $\mathrm{Im}\, W_2^{rt}$ ($\mathrm{Im}\, W_2$ in the one-particle case) contribute only for transversely polarized beams. Thus, to observe these structure functions, it is imperative to have transverse polarization, at least of one beam. A further point to notice about the contribution of $\mathrm{Im}\, W_2^{rt}$ is that if $g_{\mathrm{V}}^e = g_{\mathrm{V}}$ and $g_{\mathrm{A}}^e = g_{\mathrm{A}}$, the contribution vanishes. In other words, if the new physics contribution corresponds to the exchange of the same gauge boson as the SM contribution, so that the coupling at the e^+e^- vertex is the same, even though the final state may be produced through a new vertex, the contribution to the distribution is zero. Thus, in case of a neutral final state, where the SM contribution through a virtual photon vanishes at tree level, the observation of $\mathrm{Im}\, W_2^{rt}$ through transverse polarization could be used to determine the absorptive part of a loop contribution arising from γ exchange. In case of a charged-particle final state for which both Z and γ contribute, such a contribution would be sensitive to loop effects arising in both these exchanges.

5 Some specific processes

In this section, we will examine some exclusive processes that are of importance at ILC energies, which use the possibility of longitudinal and transverse polarization of either or both the beams to enhance the sensitivity to physics beyond the SM. Typically, the latter may be described in processes where the final state contains SM particles, as model-independent form factors or higher-dimensional operators. Of special interest to us are the processes $e^+e^- \to HZ$, $e^+e^- \to f\bar{f}$. These processes present an opportunity to demonstrate the efficacy of the framework studied here.

Since the final state in these processes has two particles, the processes are primarily described by the one-particle inclusive formalism of [4]. However, when one or both of these particles decay they give rise to a minimum of three particles in the final state, for which our present formalism would be applicable.

5.1 $e^+e^- \to HZ$

The process $e^+e^- \to HZ$ is an important mechanism for the production of the Higgs in SM. There have been suggestions [12, 15–20] that distributions of the Z or those of the decay products of the Z can be used to probe a Higgs boson in a multi-Higgs model. The process has been recently studied as a possible setting to study BSM interactions arising from a four-point e^+e^-HZ coupling [13]. This has also been extended in [21] to include leptonic decays of the Z. All possible interactions consistent with Lorentz invariance are written down in terms of completely general four-point interactions that characterize new physics in this process. In particular, these interactions are classified into whether or not they are chirality conserving or chirality violating. The former contains terms in the effective vertex with an odd number of Dirac γ matrices, while the latter contain an even number. The former set is given by V_i, A_i, $i = 1, 2, 3$ and the latter set is given by S_i, P_i, $i = 1, 2, 3$ where each of these form factors, taken to be independent of the Mandelstam variables s, t in the process, can be complex. In [13] the contributions of these to the differential cross section is evaluated. In [21], the differential cross section after the inclusion of Z decay into a pair of leptons $\ell\bar{\ell}$, where ℓ is different from e, is evaluated. We treat these two cases separately.

Taking up the the process with Z in the final state as discussed in [13], it may be readily observed that in this process, all the V_i, A_i contribute to the transverse polarization cross sections. One may immediately conclude from this observation that the spin correlations these generate are analogous to those generated by W_2 by inspection of tables in [4]. A further inspection of the tables will reveal that if the real part of a certain V_i or A_i contributes to the longitudinal cross section, then the imaginary part will not, and vice versa. This is borne out by the explicit expressions given in [13]. Finally, this does not preclude the possibility that some of the V_i or A_i will not generate spin-momentum correlations of the type generated by W_1. In the present case, the study of the explicit results of [13] reveals that it is only V_1 and A_1 that also generate spin-momentum correlations of the type generated by W_1. The S_i, P_i, on the other hand, as expected, contribute only to the transversely polarized cross section in accordance with our tables. Thus we have concretely illustrated how the general formalism that we have considered here leads to some insights and provides consistency checks on specific processes.

The comments made in Sect. 4 regarding the new information about structure functions available from polarization have to be interpreted carefully in this case. The reason is that the formalism we use in this paper assumes an s-channel exchange of a new particle. On the other hand, [13] deals with contact interactions. Hence in constructing the BSM tensors in [13], use has been made of leptonic momenta, which we do not do here.

In the extension of [13] which includes a leptonic decay of Z [21], in addition to the features discussed above, the new feature is the existence of an additional structure function, viz., W_4. This structure function appears with the vector triple product $\vec{p}_1 \times \vec{p}_2 \cdot \vec{p}_+$, which is not possible unless two independent momenta are measured in the final state. This term, for the case of a $H + (Z \to \ell^+\ell^-)$ in the final state, is CP and T odd, and is nonvanishing even in the absence of polarization. Such a term would accompany CP-violating form factors, viz., V_3 and A_3 of [13, 21], and being CPT even, would be associated with the real parts of these form factors. The correlation $\vec{p}_1 \times \vec{p}_2 \cdot \vec{p}_+$ with unpolarized and longitudinally polarized beams has indeed been studied in [21] as measure of CP violation.

A more direct application of our formalism would be to the case when only a new ZZH vertex, rather than a contact e^+e^-HZ vertex, is assumed [12, 15, 19, 20]. In this case, there are in general three independent couplings, a_Z, b_Z and $\tilde{b}_Z$, and there exist relations between the structure functions considered in case of contact interactions [13, 21] and these couplings. The observation made earlier, regarding the vanishing of the contribution of $\mathrm{Im}\, W_2^{rt}$ when only the SM gauge boson is exchanged in new-physics process, now implies that if only a new ZZH vertex, rather than contact interaction is assumed, there will be no contribution corresponding to the $\mathrm{Im}\, W_2^{rt}$ term. Alternatively, such a term will be nonvanishing for a γZH vertex, and the observation of the corresponding transverse polarization dependence would signal such a γZH vertex with an absorptive part.

5.2 $e^+e^- \to f\bar{f}$

In this section we study the process $e^+e^- \to f\bar{f}$, where f is a quark or a lepton, a process which will dominate at the ILC. We also look at further decays of the final-state fermions when they are heavy and the momentum correlations amongst these as probes of BSM interactions. We concentrate on CP-odd correlations which indicate CP violation and are therefore important to study. However, these are by no means the only interesting correlations. CP-even correlations could be used to study new CP-conserving interactions like magnetic dipole moments.

Here one may recall the one-particle inclusive case discussed in [4], where it was found that for the specific process where the final-state is a two-particle state consisting of a charged particle and its conjugate, there can be no CP-violating observables for new physics having a V, A structure in the absence of polarization. Even in the presence of transverse polarization, CP violation can be observed only in the case of scalar/pseudoscalar and tensor interactions [22]. However, CP-odd observables can be constructed if the spin of one of the final conjugate pair of particles is observed. In the case where one of the decay products is observed, the polarization of the decaying particle is being made use of indirectly. Thus it is expected that in our analysis of a two-particle inclusive states, it would be possible to incorporate CP-violating correlations.

Some years ago Hoogeveen and Stodolsky [23] considered the possibility of the observation of the electric dipole moment (EDM) of the electron in high-energy e^+e^- reactions with transversely polarized beams. Although there are stringent bounds on the magnitude of this observable today, it is interesting to recall the consequences of their very general arguments. It was pointed out that the existence of a transverse polarization vector could be used to construct two CP-odd variables, that involve (a) a scalar product with the three momentum of the particle f, and (b) a triple product involving these two momenta and the beam direction. An inspection of our tables immediately reveals that such CP-odd spin-momentum correlations occur in the presence of the form factor $g_T PF_1$.

On the other hand, as the experimental constraints on heavier SM fermion EDM's such as the τ lepton and the top quark are less stringent, there has been considerable work in trying to probe these quantities with theorists proposing several tests. Early work in this regard was the study by Couture [24, 25]. Since these particles are highly unstable, they decay very rapidly, the τ often into a ρ or a π along with neutrino emission, while the top decays into a bW. Therefore, what one considers in practice is the possibility of probing the EDM via momentum correlations of the decay products, which in reality probes the spin correlations of the fermion pair $f\bar{f}$ produced in the reaction, as the decay is due to the weak interaction which serves as a spin analyzer. The subject was studied in detail in [26] where tensor correlations constructed from the momenta of the decay products were used as a probe, following the work done in the context of the weak dipole moment, the analogue of the EDM when the photon is replaced by the Z boson, see [27]. That the sensitivity of vector correlations to weak-dipole EDM's is enhanced in the presence of longitudinal polarization was shown in [28, 29].

Here we note that the EDM of f, the fermion being pair produced, involves a coupling to the initial-state e^+e^- which is vector or axial-vector in nature, in contrast to the EDM of the electron which involves a tensor coupling. Restricting to the case of the final state being produced through V, A interactions with the initial-state e^+e^-, it is possible to find terms in Table 2, using suitable choices of vectors r, t, u and v, which correspond to CP-violating observables. Thus, for example, choosing the two observed particles to be conjugates of each other, it is possible to generate T-odd terms associated with W_2^{rt}, where one of r, t is n, and the other is p_1 or p_2, a combination of which would also be CP odd. The observable associated with W_4 is already CP and T odd. A combination of W_3^{uv} for suitable choice of u and v can easily be made CP odd (though even under T). Thus, a number of CP-odd terms can be generated. In the case of the process under consideration, restricting to γ and Z exchanges alone, these would arise from the EDM and weak dipole moment of f.

For unpolarized beams, an explicit expression for the differential cross section is provided for the case of $e^+e^- \to \tau^+\tau^- \to a(p_1)\bar{b}(p_2)X$ in [26], see (5.2) therein, for the contribution of the EDM to the differential cross-section.

What is of relevance for us here is that this expression contains scalar products of the beam direction with the sum and the cross-product of the three momenta of $a, \bar{b}$, as well as a product of each of these with the scalar product of the beam direction with the difference of the three momenta. Each of these quantities is generated from our tables as just described.

The generalization to include longitudinal polarization, derived in [28, 29], where it was emphasized that *CP*-odd vector correlations, which are suppressed in the unpolarized case, are enhanced with the use of longitudinal beam polarization. This feature can be seen from our tables, where the vector correlations are associated with the W_3 and W_4 terms. In these terms, the unpolarized correlation is necessarily proportional to g_V or g_V^e, which is numerically small. With longitudinal polarization, the polarization dependent terms involve g_A or g_A^e, providing an enhancement. For a review for these effects at high energy e^+e^- colliders, see [30] and references therein.

The work on *CP*-odd correlations with longitudinal beam polarization in τ-pair production [28, 29] was subsequently extended to the process $e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$ [31], where analogous correlations were studied. The forms of angular distributions derived in further work on the study of dipole moments through charged-lepton [32–44] and b-quark [38–45] single-particle distributions could also be deduced through the tables of our earlier work [4]. Other work where other *CP*-odd spin and momentum correlations have been studied can be found in [46].

This example has some common features with the example of $e^+e^- \to t\bar{t}$ through contact interactions discussed in [22] and through leptoquark exchange, discussed in [47].

6 Conclusions and discussion

To recapitulate, we have computed two-particle angular distributions in e^+e^- collisions arising from the interference between the virtual γ and Z exchange SM amplitudes and BSM amplitudes characterized by their Lorentz signatures, with the unknown physics lumped into structure functions. Transverse and longitudinal beam polarizations are explicitly included. We have presented a discussion on the nature of the correlations and the deductions that can be made on their polarization dependence. We have also discussed the CP and CPT properties of certain structure functions. We then specialized to specific final states HZ and $f\bar{f}$. In case of the HZ final state we find some subtle effects that are absent when only one-particle inclusive processes are considered. A summary of a variety of effects due to popular sources of BSM physics as manifested in the present framework is provided. Our work demonstrates the power of the general model-independent framework and justifies the extension of results known for one-particle distributions to two-particle distributions.

As discussed in Sect. 1, processes which involve t- and u-channel contributions either at the level of SM or in the new physics, strictly speaking, lie outside the scope of the present formalism. In those cases, not only do the structure functions we use depend on additional invariants involving the electron and positron momenta, even the tensors written down for the interference of the SM terms with the BSM terms would involve the electron and positron momenta. This requires further study. Such a formalism when developed would find application to several processes where t- or u-channel exchanges could contribute to the SM amplitude, as for example in W^+W^- production, wherein transverse-polarization effects were studied in [48–50], or in γZ production, polarization effects in which were studied in [7–9]. An extension of such a formalism would be useful even when considering interference effects between s-channel and t-channel exchanges in a final state like a pair of neutralinos in the supersymmetric extension of SM. Such a process was studied for example in [51]. Another situation where an extended formalism would be useful is when the new physics is expressed as four-point contact interactions, as for example in [10, 13, 21].

Another class of interesting processes involve BSM spectrum of particles, as for example in a theory like the minimal supersymmetric standard model (MSSM). A description of such processes would also involve a modification of our formalism, not only to include new particles, but also to include t- and u-channel exchanges.

Some popular scenarios, such as extra-dimensional models, non-commutative models, contact interactions, etc., could also be studied with a more general formalism indicated. One would expect to make some general predictions in these cases.

Even though Dass and Ross in their paper [6] discuss the one-particle inclusive process where the spin of the observed particle is also measured, we have not treated this topic in our context in this work. This would be an interesting future study. In one of the examples we consider, the final state arises from the decay of a real or virtual spin-1 state. Hence the analysis presented in [6], which was for a decaying spin-$\frac{1}{2}$ particle, is not directly applicable. It would be useful to extend the formalism of [6] to spin 1.

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References

1. A. Djouadi, J. Lykken, K. Monig, Y. Okada, M.J. Oreglia, S. Yamashita et al., arXiv:0709.1893 [hep-ph]
2. G.A. Moortgat-Pick et al., Phys. Rep. **460**, 131 (2008) [arXiv:hep-ph/0507011]
3. P. Osland, N. Paver, In: Proc. of 2005 International Linear Collider Workshop (LCWS 2005), Stanford, California, 18–22 March 2005, pp. 0301 [arXiv:hep-ph/0507185]
4. B. Ananthanarayan, S.D. Rindani, Eur. Phys. J. C **46**, 705 (2006)
5. G.V. Dass, G.G. Ross, Phys. Lett. B **57**, 173 (1975)
6. G.V. Dass, G.G. Ross, Nucl. Phys. B **118**, 284 (1977)

7. B. Ananthanarayan, S.D. Rindani, R.K. Singh, A. Bartl, Phys. Lett. B **593**, 95 (2004) [arXiv:hep-ph/0404106]
8. B. Ananthanarayan, S.D. Rindani, R.K. Singh, A. Bartl, Phys. Lett. B **608**, 274 (2005) [Erratum]
9. B. Ananthanarayan, S.D. Rindani, JHEP **0510**, 077 (2005) [arXiv:hep-ph/0507037]
10. B. Ananthanarayan, S.D. Rindani, Phys. Lett. B **606**, 107 (2005) [arXiv:hep-ph/0410084]
11. S.D. Rindani, Pramana **45**, S263 (1995) [arXiv:hep-ph/9411398]
12. K. Hagiwara, M.L. Stong, Z. Phys. C **62**, 99 (1994) [arXiv:hep-ph/9309248]
13. K. Rao, S.D. Rindani, Phys. Lett. B **642**, 85 (2006) [arXiv:hep-ph/0605298]
14. S. Lakshmibala, G. Rajasekaran, S.D. Rindani, Phys. Lett. B **117**, 345 (1982)
15. G.J. Gounaris, F.M. Renard, N.D. Vlachos, Nucl. Phys. B **459**, 51 (1996) [arXiv:hep-ph/9509316]
16. Q.H. Cao, F. Larios, G. Tavares-Velasco, C.P. Yuan, Phys. Rev. D **74**, 056 001 (2006) [arXiv:hep-ph/0605197]
17. V. Barger, T. Han, P. Langacker, B. McElrath, P. Zerwas, Phys. Rev. D **67**, 115 001 (2003) [arXiv:hep-ph/0301097]
18. W. Kilian, M. Kramer, P.M. Zerwas, arXiv:hep-ph/9605437
19. T. Han, J. Jiang, Phys. Rev. D **63**, 096 007 (2001) [arXiv:hep-ph/0011271]
20. S.S. Biswal, R.M. Godbole, R.K. Singh, D. Choudhury, Phys. Rev. D **73**, 035 001 (2006) [arXiv:hep-ph/0509070]
21. K. Rao, S.D. Rindani, Phys. Rev. D **77**, 015 009 (2008) [arXiv:0709.2591 [hep-ph]]
22. B. Ananthanarayan, S.D. Rindani, Phys. Rev. D **70**, 036 005 (2004) [arXiv:hep-ph/0309260]
23. F. Hoogeveen, L. Stodolsky, Phys. Lett. B **212**, 505 (1988)
24. G. Couture, Phys. Lett. B **272**, 404 (1991)
25. G. Couture, Phys. Lett. B **305**, 306 (1993)
26. W. Bernreuther, O. Nachtmann, P. Overmann, Phys. Rev. D **48**, 78 (1993)
27. W. Bernreuther, U. Low, J.P. Ma, O. Nachtmann, Z. Phys. C **43**, 117 (1989)
28. B. Ananthanarayan, S.D. Rindani, Phys. Rev. Lett. **73**, 1215 (1994) [arXiv:hep-ph/9310312]
29. B. Ananthanarayan, S.D. Rindani, Phys. Rev. D **51**, 5996 (1995) [arXiv:hep-ph/9411399]
30. B. Ananthanarayan, S.D. Rindani, A. Stahl, Eur. Phys. J. C **27**, 33 (2003) [arXiv:hep-ph/0204233]
31. F. Cuypers, S.D. Rindani, Phys. Lett. B **343**, 333 (1995) [arXiv:hep-ph/9409243]
32. P. Poulose, S.D. Rindani, Phys. Lett. B **383**, 212 (1996) [arXiv:hep-ph/9606356]
33. P. Poulose, S.D. Rindani, Phys. Rev. D **54**, 4326 (1996) [arXiv:hep-ph/9509299]
34. P. Poulose, S.D. Rindani, Phys. Rev. D **61**, 119 901 (2000) [Erratum]
35. P. Poulose, S.D. Rindani, Phys. Lett. B **349**, 379 (1995) [arXiv:hep-ph/9410357]
36. S.D. Rindani, Pramana **61**, 33 (2003) [arXiv:hep-ph/0304046]
37. S.D. Rindani, Pramana **54**, 791 (2000) [arXiv:hep-ph/0002006]
38. B. Grzadkowski, Z. Hioki, Phys. Lett. B **557**, 55 (2003) [arXiv:hep-ph/0208079]
39. B. Grzadkowski, Z. Hioki, Phys. Lett. B **529**, 82 (2002) [arXiv:hep-ph/0112361]
40. B. Grzadkowski, Z. Hioki, Nucl. Phys. B **585**, 3 (2000) [arXiv:hep-ph/0004223]
41. B. Grzadkowski, Z. Hioki, Phys. Lett. B **476**, 87 (2000) [arXiv:hep-ph/9911505]
42. B. Grzadkowski, Z. Hioki, Phys. Rev. D **61**, 014 013 (2000) [arXiv:hep-ph/9805318]
43. B. Grzadkowski, Z. Hioki, arXiv:hep-ph/9610306
44. B. Grzadkowski, Z. Hioki, M. Szafranski, Phys. Rev. D **58**, 035 002 (1998) [arXiv:hep-ph/9712357]
45. E. Christova, D. Draganov, Phys. Lett. B **434**, 373 (1998) [arXiv:hep-ph/9710225]
46. D. Atwood, S. Bar-Shalom, G. Eilam, A. Soni, Phys. Rep. **347**, 1 (2001) [arXiv:hep-ph/0006032]
47. S.D. Rindani, Phys. Lett. B **602**, 97 (2004) [arXiv:hep-ph/0408083]
48. J. Fleischer, K. Kolodziej, F. Jegerlehner, Phys. Rev. D **49**, 2174 (1994)
49. M. Diehl, O. Nachtmann, F. Nagel, Eur. Phys. J. C **32**, 17 (2003) [arXiv:hep-ph/0306247]
50. M. Diehl, O. Nachtmann, F. Nagel, Eur. Phys. J. C **27**, 375 (2003) [arXiv:hep-ph/0209229]
51. S.Y. Choi, M. Drees, J. Song, JHEP **0609**, 064 (2006) [arXiv:hep-ph/0602131]